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MIXING OF A CONTACT BOUNDARY RETARDED BY STATIONARY SHOCK WAVES

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The phenomenon of turbulent mixing of the interface between two gases of different densities retarded by plane stationary shock waves moving from the light gas into the heavy one was discovered experimentally in [1].

It is shown below that within the framework of the semiempirical models of [1-3] this phenomenon is determined by the size of the initial perturbations – the roughness of the interface. If the characteristic size of these perturbations approaches zero, then the width of the mixing region also approaches zero. This phenomenon is explained by the δ-function character of the acceleration.

If the acceleration varies smoothly, such as constantly, then mixing will always develop, even with infinitely small roughness. The analytical dependence of the width of the mixing region on the initial roughness is presented.

The interface of the gases (liquids) is unstable against small perturbations if the acceleration is directed from the light to the heavy gas. This instability develops for sufficiently small coefficients of viscosity and surface tension.

In the semiempirical models of [1-3] it is assumed that turbulent mixing develops simultaneously with the action of the acceleration, although actually the presence of viscosity and surface tension leads to the appearance of a finite time interval during which a gradual transition to turbulent motion occurs.

The known self-similar solutions [3-5] were obtained under the assumption of smallness of the initial perturbations. Actually, these perturbations may not be small. The law according to which the emergence into a self-similar solution with constant acceleration occurs is established below. A mild "forgetting" of the initial irregularities of the surface was unexpectedly discovered.

1. Approximate Model

We will consider a diffusional model of turbulent mixing in the approximate formulation of [5]: The fluids are incompressible, while the turbulent velocity v is assumed to be a function of time only. Then the process of turbulent mixing will be described by two equations for two unknowns (the density ρ of the mixture and the characteristic turbulent velocity v),

$$\frac{\partial \rho}{\partial t} = lv \frac{\partial^2 \rho}{\partial x^2}; \qquad (1.1)$$

$$\frac{1}{2}\frac{dv^2}{dt} + \frac{\nu}{\alpha}\frac{v^3}{L} = \alpha v \omega^2, \qquad (1.2)$$

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where L is the effective width of the mixing region, equal to the distance between the points at which the dimensionless density $\delta = (\rho - \rho_2)/(\rho_1 - \rho_2)$ takes values of 0.1 and 0.9; ρ_1 and ρ_2 are the densities of the light and heavy fluids, so that for the density of the mixture there is the equality $\rho = \rho_1 + \rho_2$; ν is a second empirical constant; $\overline{\omega^2}$ is the expression $g\partial \ln\rho/\partial x$ averaged over the mixing region; g is the acceleration. The concentration of the light fluid is expressed through the density ρ,

$$c = \frac{\rho_2}{\rho} = \frac{\rho_2^0(\rho - \rho_2^0)}{\rho(\rho_1^0 - \rho_2^0)},$$

and therefore it does not take part in the further discussion.

It is assumed that the characteristic turbulent length, appearing in the coefficient of diffusion, is connected with the mixing region by some constant α, determined from experiment:

$$l = \alpha L.$$

We will assume that at the initial time the interface is the point $x = 0$. Since ρ_1^0 and ρ_2^0, the initial densities of the heavy and light fluids, are constants while l and v are functions of time only, Eq. (1.1) is transformed by a change of variables to the equation of diffusion with a constant coefficient [5],

$$\partial\rho/\partial\tau = \partial^2\rho/\partial x^2, \tag{1.3}$$

where $d\tau = l v dt$. The solution of Eq. (1.3) can be represented through the probability integral, while the following expression is valid for the width L:

$$L = 4\eta_0\tau^{1/2}, \quad \eta_0 = 0.906. \tag{1.4}$$

Eliminating τ from (1.4), we obtain the equation

$$\frac{dL}{dt} = 8\eta_0\alpha v. \tag{1.5}$$

2. Dependence on Initial Data with Constant Acceleration

Equations (1.2) and (1.5) with arbitrary initial data L_0 and v_0 are subject to investigation.

First, let us be confined to the case of $v_0 = 0$. We convert Eq. (1.2) to the form

$$\frac{dv}{dt} = \alpha A_1 - \frac{\nu}{\alpha}\frac{v^2}{L}, \tag{2.1}$$

where A_1 is some constant determined by the initial data. From (1.5) and (2.1) we get the equation

$$\frac{dL}{dv} = \frac{8\eta_0^2\alpha^2 vL}{\alpha^2 A_1 L - \nu v^2}.$$

Its solution has the form

$$v^2 = \frac{A_1\alpha^2}{4\alpha^2\eta_0^2 + \nu}L + cL^{-\nu/4\eta_0^2\alpha^2}, \tag{2.2}$$

where c is an arbitrary constant. With zero initial data $L_0 = v_0 = 0$ the constant c equals zero, while the substitution of (2.2) into (1.5) and subsequent integration lead to the well-known solution

$$L = 16\eta_0^4\alpha^4\frac{A_1}{4\eta_0^2\alpha^2 + \nu}t^2.$$

If the initial roughness is $L_0 \neq 0$ while $v_0 = 0$, then the problem is reduced to the equation

$$\frac{dL}{dt} = \frac{8\eta_0^2\alpha^2\sqrt{A_1}}{\sqrt{4\alpha^2\eta_0^2 + \nu}}\sqrt{L - L_0\left(\frac{L_0}{L}\right)^{\nu/4\eta_0^2\alpha^2}}.$$

We integrate it approximately, discarding the second term inside the radical:

$$\sqrt{L} = \frac{4\eta_0^2\alpha^2\sqrt{A_1}}{\sqrt{4\alpha^2\eta_0^2 + \nu}}t + \sqrt{L_0}. \tag{2.3}$$

We designate the solution with an initial perturbation of L_0 as L', and then the connection with the unperturbed solution L is established:

$$\sqrt{L'/L} = 1 + \sqrt{L_0/L}.$$

The mild "forgetting" of the initial data follows from this equation. Thus, a 1% roughness ($L_0/L = 0.01$) leads to a 20% departure from the exact solution ($L'/L = 1.21$).

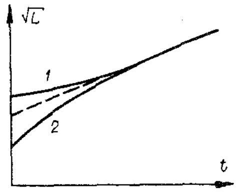

Fig. 1

We can qualitatively indicate the picture of the emergence into a self-similar mode in the general case of nonzero initial data v_0 and L_0. From (1.5) and (2.1) we obtain the initial value of the second derivative of the function $\sqrt{L}$:

$$\left.\frac{d^2\sqrt{L}}{dt^2}\right|_{t=0} = -\left(16\eta_0^4\alpha^2 + 4\eta_0^2\nu\right)\frac{v_0^2}{L_0} + \frac{4\eta_0^2\alpha^2 A_1}{\sqrt{L_0}}.$$

If $v_0 = 0$ then $d^2\sqrt{L}/dt^2 > 0$. In this case the emergence into a linear law in the variables $\sqrt{L}$ and t occurs with a delay (Fig. 1, curve 1).

If $v_0 \neq 0$ then the case when $d^2\sqrt{L}/dt^2 < 0$ is possible. Then more rapid development initially occurs with subsequent slowing and emergence into a linear law (Fig. 1, curve 2).

The dashed straight line in Fig. 1 corresponds to the approximate solution (2.3).

3. The δ-Function Form of Acceleration

This case models the passage of a stationary shock wave through a contact boundary. Let us study the dependence on the initial data. We will consider the case when the boundary has a certain roughness L_0 before the arrival of the shock wave. The action of the shock wave leads to the fact that the initial data for the turbulent velocity become nonzero.

Actually, by integrating the initial equation (1.2) over time and taking the upper integration limit to zero, we obtain

$$v_0 = \alpha U_0 A_2, \tag{3.1}$$

where U_0 is the velocity of the boundary after the passage of the shock wave; A_2 is a constant. Then Eq. (1.2) must be taken with a zero right side and with the initial data (3.1):

$$\frac{dv}{dt} + \frac{\nu}{\alpha}\frac{v^2}{L} = 0. \tag{3.2}$$

The two equations (1.5) and (3.2) are reduced to one:

$$\frac{dL}{dv} = -\frac{8\alpha^2\eta_0^2}{\nu}\frac{L}{v}.$$

Its solution has the form

$$L = L_0\left(\frac{v_0}{v}\right)^{8\alpha^2\eta_0^2/\nu}.$$

From the latter equation it follows that a nonzero solution is possible only for $L_0 \neq 0$. The law of development of the width of the mixing region is determined after integration of Eq. (1.5):

$$L = L_0^{\frac{\nu}{\nu+8\alpha^2\eta_0^2}}\left(\frac{\nu + 8\alpha^2\eta_0^2}{\alpha}v_0 t + L_0\right)^{\frac{8\alpha^2\eta_0^2}{\nu+8\alpha^2\eta_0^2}}. \tag{3.3}$$

The exponent is always less than unity. Only a trivial solution is possible when $L_0 = 0$.

4. Comparison with the Results of Other Reports

Turbulent mixing of air with helium was studied in [1]. The gases were initial separated by a thin film and then a stationary shock wave with a Mach number M = 1.3 passed through the boundary, entered the helium, was reflected from a rigid wall, and again arrived at the boundary. After reflection from it, and then from the rigid wall again, the shock wave repeatedly retarded the contact boundary. Turbulent mixing of the interface was observed experimentally.

We made a numerical calculation of this problem. We used the following system of equations:

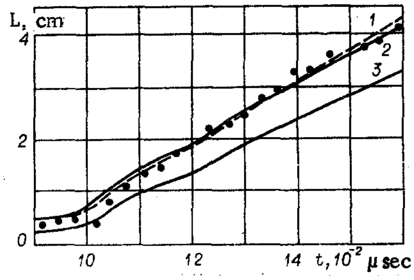

Fig. 2

$$d\rho/dt + \rho\,\partial u/\partial x = 0 \text{ — the continuity equation;} \quad (4.1)$$

$$du/dt = -\partial p/\rho\,\partial x \text{ — the equation of motion;} \quad (4.2)$$

$$dc/dt = -\partial j/\rho\,\partial x,\ j = -\rho l v \partial c/\partial x \text{ — the equation for the concentration;} \quad (4.3)$$

$$\frac{d\varepsilon}{dt} + p\frac{d(1/\rho)}{dt} = -\frac{\partial q}{\partial x},\quad q = -\rho l v\left(\frac{\partial \varepsilon}{\partial x} + p\frac{\partial(1/\rho)}{\partial x}\right) \text{ — the energy equation;} \quad (4.4)$$

$$\frac{d\rho v^2}{dt} + \nu\frac{\rho v^3}{l} = \rho l v \omega^2 + \beta\frac{\partial}{\partial x}\left(\rho l \frac{\partial v^3}{\partial x}\right), \quad (4.5)$$

where $\omega^2 = g(\partial\rho/\rho\,\partial x + g/a^2)$; $g = -\partial p/\rho\partial x$; a is the speed of sound; $p = p_1(1-c) + p_2 c$; $\varepsilon = \varepsilon_1(1-c) + \varepsilon_2 c$; $\beta = 0.5$.

This model of turbulent mixing differs from that of [2] by the introduction of Eq. (4.5) for the turbulent velocity v. This change in the model permits the description of non-self-similar problems with sharply varying acceleration.

A comparison with the model of [1] shows that the models coincide in basic features. The differences pertain to unimportant terms, as well as to the empirical constants chosen. This is unimportant for the results of the present report.

Equations (4.1)-(4.5) were replaced by difference equations through the scheme suggested in [2]. The method of calculating the width of the mixing region was taken from [2].

The problem under consideration proved to be very sensitive to the spatial grid. This is explained by the method of calculating the width of the mixing region. In the method it is such that the initial width has a nonzero value on the order of a grid interval. More precisely,

$$L_0 = \frac{1}{2}(h_1 + h_2),$$

where h_1 and h_2 are the steps in space to the left and right of the interface.

With smoothly varying acceleration the initial width can be calculated from an analytical equation, as in [2], but such a method is inadmissible in the present case. Therefore, the convergence was traced by subdividing the grid intervals in the region of the boundary. It was found, as in the theoretical analysis in Sec. 3, that with a small initial width L_0 there is little mixing.

The experimental results of [1] are shown by points in Fig. 2. In the experiment a perturbation region of $L_0 \approx 5$ mm was observed by the time of the first retardation, t = 1000 μsec. It follows from Eq. (3.3) that agreement with the experiment can be achieved through the appropriate choice of α and ν with a fixed L_0. Satisfactory agreement with the experiment was obtained, for example, with $\alpha = 0.31$ and $\nu = 1.25$ (curve 2), as well as with $\alpha = 0.25$ and $\nu = 0.3$ (curve 1). Curve 3 corresponds to the constants of curve 2 and $L_2 = 2.5$ mm.

In the calculations of [1] the turbulent mixing was turned on at the moment of the first retardation. The initial width of the mixing region was nonzero and determined all the subsequent process.

Thus, the models of turbulent mixing in [1] and in the present report are such that the width of the mixing region developing under the action of a series of stationary shock waves is fully determined by the characteristic size of the initial perturbation.

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PRESSURE PULSATIONS IN A RECESS OVER WHICH A SUBSONIC OR SUPERSONIC GAS STREAM FLOWS

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Subsonic or supersonic flow over a recess of small depth with laminar, transitional, and turbulent modes of flow in the boundary layer ahead of the separation point are discussed. Under certain conditions (the Mach number of the external stream, the size and shape of the recess, etc.) discrete components are observed in the spectrum of pressure pulsations of the recess. This phenomenon has been investigated both experimentally [1-4] and theoretically [5] with turbulent flow in the boundary layer ahead of the recess. In [1, 4] the nonsteady flow pattern in the vicinity of a recess was revealed mainly using shadow photographs obtained with a short exposure ($\sim 10^{-6}$ sec). The frequencies of the discrete components in the pressure spectrum of a three-dimensional rectangular recess were determined in [3].

In the present report we investigate in detail the nonsteady flow pattern in a recess and its vicinity with laminar, transitional, and turbulent modes of flow in the boundary layer ahead of the recess. It is shown that with laminar flow in the boundary layer a nonsteady separation zone of small size, which periodically disappears and reappears, forms ahead of the recess because of the pressure pulsations in it. The compression shocks formed ahead of this zone of separation flow and the vortices formed in the zone are periodically carried off by the stream after the next disappearance of this zone.

1. The experimental investigation was performed in wind tunnels and on an aeroballistic course. The test models comprise two groups. The first group (I) includes cones with half-angles $\theta = 2.5$-$30°$ at the apex and with axisymmetric annular recesses on the lateral surface with a depth h and a relative length $l^0 = l/h$. The second group (II) includes flat plates, which comprised a side wall of the working section of the wind tunnel with a size of 70×50 mm. Recesses with a depth h, a relative length l^0, and a width of 70 mm were made in these plates. A capacitive detector of pressure pulsations was mounted on the bottom of the recess flush with its surface. The frequency characteristic curve of the detector has a maximum at the frequency f = 6.5 kHz, which corresponds to the natural frequency of the detector membrane. Therefore, all the measurements were made at frequencies f < 6 kHz. The parameters of the stream and the dimensions of the models used in the experiments are given in Table 1, where M_0 is the Mach number of the oncoming stream; T_W/T_0 is the ratio of the wall temperature to the stagnation temperature in the outer stream; α_1 and α_2 are the angles between the leading or trailing walls of the recess and its bottom; $z^0 = z/h$ is the relative length of the deflector; d is the diameter of the midsection of the model; Re is the Reynolds number, calculated from the parameters of the outer stream and the length of the model from the critical point to the leading edge of the recess; A and B are the groups of experiments conducted on the aeroballistic course and in wind tunnels, respectively. The parameters of the stream on the aeroballistic course were determined from the average velocity over a base with a length of 8 m; the error in measuring this velocity did not exceed 0.5%.

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